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## SEISMIC RESPONSE OF FLEXIBLE WALLS RETAINING HOMOGENEOUS VISCOELASTIC SOIL

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### ABSTRACT

A simplified analytical solution is derived for the dynamic response of a flexible vertical retaining wall supported on a rotationally compliant footing, subjected to vertically-propagating harmonic S-waves under plane-strain conditions. The wall retains a semi-infinite, homogeneous viscoelastic soil layer of constant thickness and material properties. The proposed solution is based on the Veletsos-Younan simplifying assumption of zero vertical normal stresses in the soil, and negligible variation of vertical displacements with horizontal distance from the wall. A modified integration technique is employed, inspired by the seminal work of Vlasov and Leontiev, which simplifies the analysis by suppressing the vertical coordinate and transforming the governing partial differential equation into an ordinary one that admits an elementary solution. Both cantilever and top-hinged walls are studied. Closed-form solutions are derived for lateral soil displacements, dynamic soil pressures, and equivalent Winkler springs connecting the wall to the far-field soil. It is shown that for cantilever conditions even a small amount of wall flexibility leads to a strong reduction in soil thrust, while the rotation at the wall base causes an additional decrease in thrust. The predictions of the method are in good agreement with available solutions, while new results for combined wall flexibility and rotational compliance are presented. The proposed approach offers a simpler alternative to the complex elastodynamic solutions of Veletsos and Younan.

*Keywords: Flexible retaining wall; Dynamic response; Soil-structure interaction*

### 1. INTRODUCTION

A considerable amount of research has been devoted in exploring the seismic response of retaining structures by properly accounting for soil-structure interaction and wave propagation effects. The first dynamic solutions were developed by Matsuo & Ohara (1960) and Tajimi (1969), under the simplifying assumption of zero vertical dynamic movements in the backfill under horizontal ground excitation and plane strain conditions. These early models were followed by an exact elastodynamic solution developed for a pair of rigid walls by Wood (1973), and a simplified Winkler model for single walls by Scott (1973).

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A second generation of elastodynamic models was proposed by Arias et al (1981), Veletsos & co-workers (Veletsos & Younan 1994, Younan & Veletsos 2000), Li (1999), Wu & Finn (1999), and Beskos & co-workers (Theodorakopoulos et al 2001, 2004), which extended earlier studies to cover various configurations including flexible and rotationally constrained walls at the base, cylindrical vaults, top-hinged walls, elastic bedrock, and inhomogeneous or poroelastic soil. These seminal studies were followed by a number of exact solutions (extensions of Wood's solution) developed for a pair of walls encompassing different boundary conditions (Papazafeiropoulos & Psarropoulos 2010), poroelastic material behaviour (Papagiannopoulos et al 2015) and variation of material properties with depth (Vrettos et al 2016). A simplified variant of the above studies, which allows closed-form solutions to be obtained for rigid single walls, rotationally constrained single walls, and pairs of walls with corresponding properties, was proposed by Kloukinas et al (2012).

Despite their idealised nature, the above solutions have succeeded in implicitly demonstrating that: (1) earth thrusts are inherently linked to dynamic soil and wall displacement response - not to body forces imposed on a soil wedge behind the wall, like those assumed in the classical pseudo-static limit state models by Okabe (1924), Mononobe & Matsuo (1929) and their variants (e.g., Seed & Whitman 1970; Richards & Elms 1979; Steedman & Zeng 1989, Mylonakis et al 2007); (2) soil pressures strongly depend on wall flexibility and kinematic constraints, and may significantly exceed those predicted by limit state solutions; (3) soil pressures strongly depend on frequency – or, equivalently, seismic wavelength – due to backfill dynamics; (4) the point of application of the overall seismic thrust is frequency dependent, a behaviour which cannot be captured by limit state solutions. Nevertheless, the above studies were traditionally viewed by engineers merely as “elastic alternatives” to the established limit state (“plastic”) solutions. The fundamental difference between displacement- and force-generated soil thrusts was not explicitly recognised as a separate analysis framework.

Recently, Brandenburg et al (2015, 2017a,b), proposed a new engineering-oriented analysis framework which treats seismic earth-pressures as a kinematic interaction problem associated with 1D wave propagation in the backfill and the wall foundation, analogous to that employed in dynamic analysis of piles and embedded footings. Their approach is applied to walls that are not resting on a rigid base and accounts for soil inhomogeneity and various kinematic constraints. Other work adopting a conceptually similar approach has been presented by Davis (2003) and Vrettos and Feldbusch (2017).

The objective of this paper is to further explore the importance of wall flexibility, rotational compliance at the base and kinematic constraints, by extending the simplified model of Kloukinas et al (2012) to account for these effects. It is shown that simple wave solutions are still possible under such conditions, which can provide valuable insight into the physics of the problem. It is worth noting that the objectives of the paper are similar to those of a recent paper by several of the authors (Brandenburg et al 2017b), yet the analysis approach and some of the assumptions involved are different.

## 2. STATEMENT OF PROBLEM

### 2.1 Equation of motion for the soil

The problem under consideration is depicted in Figure 1, where Cartesian coordinates are employed. A vertical elastic wall of height  $H$ , thickness  $t_w$ , bending stiffness  $EI$ , mass density  $\rho_w$ , Poisson's ratio  $\nu_w$  and damping coefficient  $\delta_w$ , retains a semi-infinite homogeneous horizontal soil stratum of constant thickness  $H$ , mass density  $\rho$ , Poisson's ratio  $\nu$ , shear modulus  $G$  and damping coefficient  $\delta$ . The solution is developed in the frequency domain considering a rock motion of frequency  $\omega$  and amplitude  $\ddot{X}_g$ . This leads to vertical S-wave propagating from bedrock to ground surface. Different boundary conditions at the wall top are considered, including cantilever (Figure 1a) and top-hinged (Figure 1b). For propped walls, the latter conditions are admittedly incompatible with the antisymmetric nature of the earthquake excitation at hand, yet they may arise in presence of internal diaphragms as discussed in Brandenburg et al (2017b). It is worth noting that the system rests on a rigid base, which may lead to strong resonances within the backfill relative to what is usually observed in practical applications. However, this assumption is not an essential point of the analysis and may be relaxed in a straightforward manner as shown in Li (1999).

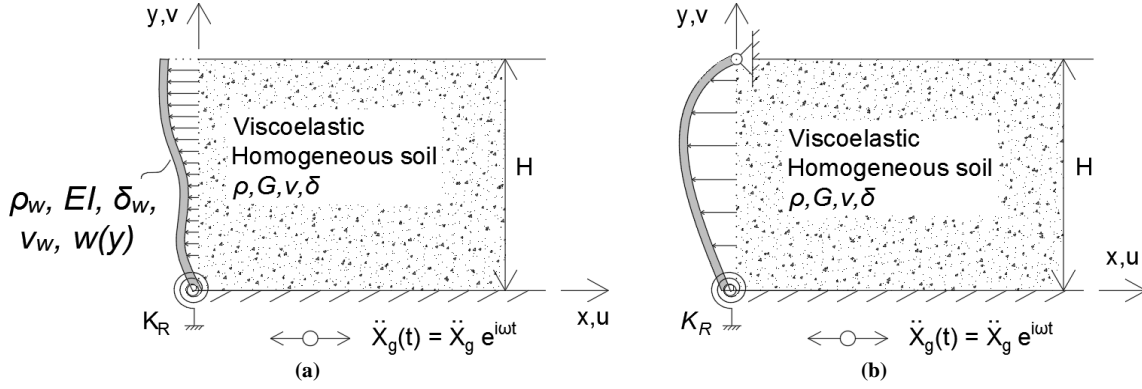


Figure 1. System of flexible wall retaining a homogeneous semi-infinite soil stratum, (a) cantilever wall and (b) top-hinged wall on a rotationally compliant footing.

Considering the equilibrium in the horizontal direction of an arbitrary piece of retained soil, leads to the differential equation:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = \rho \left( \frac{\partial^2 u}{\partial t^2} + \ddot{X}_g \right) \quad (1)$$

where  $\sigma_x$  and  $\tau_{xy}$  are the vertical and shear normal stresses and  $u$  is the horizontal displacement of the soil relative to the rigid base. For plane-strain conditions, combining the stress-strain relations for a linearly viscoelastic medium with the field equations yields (Veletsos & Younan 1994):

$$\sigma_x = \frac{E^*(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial u}{\partial x} + \frac{E^*\nu}{(1+\nu)(1-2\nu)} \frac{\partial v}{\partial y} \quad (2)$$

$$\sigma_y = \frac{E^*\nu}{(1+\nu)(1-2\nu)} \frac{\partial u}{\partial x} + \frac{E^*(1-\nu)}{(1+\nu)(1-2\nu)} \frac{\partial v}{\partial y} \quad (3)$$

$$\tau_{xy} = G^* \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \quad (4)$$

where  $E^* = E(1+i\delta)$  and  $G^* = G(1+i\delta)$  are the complex Young's and shear modulus of the soil material, respectively, and  $\nu$  is the vertical displacement of the soil material relative to the base. Adopting the assumptions of Veletsos and Younan (1994) of: (1) negligible vertical dynamic normal stresses during lateral ground shaking ( $\sigma_y = 0$ ); and (2) negligible variation of vertical displacement in the horizontal direction, everywhere in the soil ( $\partial v / \partial x = 0$ ), Equation 1 can be written in the uncoupled Navier form:

$$\frac{2-\nu}{1-\nu} G^* \frac{\partial^2 u}{\partial x^2} + G^* \frac{\partial^2 u}{\partial y^2} - \rho \frac{\partial^2 u}{\partial t^2} = \rho \ddot{X}_g(t) \quad (5)$$

Employing the method of separation of variables,

$$u(x, y) = X(x) \Phi(y) \quad (6)$$

where  $X(x)$  is an unknown function of the horizontal coordinate having dimensions of length, and  $\Phi(y)$  a known dimensionless function of depth that is at least once differentiable and satisfies the essential boundary condition of zero displacement at the base. Following the modified Vlasov-Leontiev technique employed by Kloukinas et al (2012), Equation 5 is multiplied by  $\Phi(y)$ , integrated over the  $y$ -axis and then integrated by parts while enforcing the boundary condition  $\Phi'(0) = 0$  which guarantees zero shear tractions at the soil surface. This leads to the following ordinary differential equation:

$$\frac{d^2 X(x)}{dx^2} - \left( \frac{a_{oc}^2 - a_o^2}{H^2 \psi_e^2} \right) X(x) = \frac{\mathcal{L}}{\psi_e^2 V_s^{*2}} \ddot{X}_g \quad (7)$$

where

$$a_o = \frac{\omega H}{V_s^*} \quad (8a)$$

$$a_{oc}^2 = H^2 \left[ \int_0^H \left( \frac{d\Phi}{dy} \right)^2 dy / \int_0^H \Phi^2 dy \right] \quad (8b)$$

$$\mathcal{L} = \int_0^H \Phi dy / \int_0^H \Phi^2 dy \quad (8c)$$

$$\psi_e^2 = \frac{2-\nu}{1-\nu} \quad (8d)$$

Of the above coefficients,  $a_o$  is a dimensionless excitation frequency,  $a_{oc}$  is a corresponding cut-off frequency beyond which stress waves start to propagate horizontally in the retained soil mass,  $\mathcal{L}$  is a modal participation coefficient depending on the shape function  $\Phi(y)$  and  $V_s^*$  is the complex wave propagation velocity of the backfill ( $V_s^{*2} = G^*/\rho$ ). Finally,  $\psi_e$  is a compressibility coefficient dependent solely on Poisson's ratio.

The general solution to Equation 7 is:

$$X(x) = A_1 e^{mx} + A_2 e^{-mx} - \frac{\mathcal{L}}{(a_{oc}^2 - a_o^2)} \left( \frac{H}{V_s^*} \right)^2 \ddot{X}_g \quad (9)$$

where  $m = \sqrt{a_{oc}^2 - a_o^2}/H\psi_e$  is a complex wavenumber controlling the attenuation of soil motion with horizontal distance from the wall. To determine the integration constants  $A_1$  and  $A_2$ , the following boundary conditions are enforced:

$$u(x \rightarrow \infty, y) = u_{ff}(y) \quad (10a)$$

$$u(0, y) = w(y) \quad (10b)$$

where  $u_{ff}(y)$  is the free-field soil displacement developing at an infinite distance from the wall and  $w(y)$  is the deflection of the retaining structure. For a pair of flexible walls spaced at a distance  $L$ , the boundary condition in Equation 10a is replaced by  $u(L, y) = w(y)$ , which arises from antisymmetry of loading. To determine  $u_{ff}(y)$ , the arbitrary shape function  $\Phi(y)$  can be selected, as a first approximation, as the fundamental mode shape of a homogeneous soil column (Kloukinas et al. 2012):

$$\Phi(y) = \sin\left(\frac{\pi y}{2H}\right) \quad (11)$$

which satisfies the requirements of differentiability and zero displacement at the base and reproduces the natural frequency of the soil layer. It should be stressed though that it is unlikely for the wall itself to deform in the way described by  $\Phi(y)$  in Equation 11, so the particular shape is not expected to provide a good approximation to soil displacement near the wall. This has implications in the value of Winkler stiffness as discussed later in this paper.

Considering the boundary conditions in Equations 10, the displacement function  $X(x)$  and the associated soil thrust, are obtained from Equations 12 and 13, respectively, in the form:

$$X(x) = e^{-mx} \frac{w(y)}{\Phi(y)} + \frac{\mathcal{L}}{(a_{oc}^2 - a_o^2)} \left( \frac{H}{V_s^*} \right)^2 \ddot{X}_g (e^{-mx} - 1) \quad (12)$$

$$\sigma_x(0, y) = \frac{2}{1-\nu} G^* \frac{\partial u}{\partial x} \Big|_{x=0} = -\frac{2}{1-\nu} G^* m \left[ \frac{\mathcal{L}}{(a_{oc}^2 - a_0^2)} \left( \frac{H}{V_s^*} \right)^2 \ddot{X}_g \Phi(y) + w(y) \right] \quad (13)$$

## 2.2 Equation of motion for the wall

The retaining wall is modelled as an Euler-Bernoulli beam of unit width. The equilibrium of forces acting at an arbitrary segment of the wall (Figure 2) leads to the differential equation:

$$EI^* w^{(4)}(y) - \sigma_x(0, y) + \rho_w \left( \frac{\partial^2 w(y, t)}{\partial t^2} + \ddot{X}_g \right) = 0 \quad (14)$$

where  $EI^* = E_w^* t_w^3 / [12 (1 - \nu_w^2)]$ ,  $E_w^* = E_w (1 + i \delta_w)$  denote the wall bending stiffness under plane strain conditions, and the corresponding complex wall modulus, respectively.

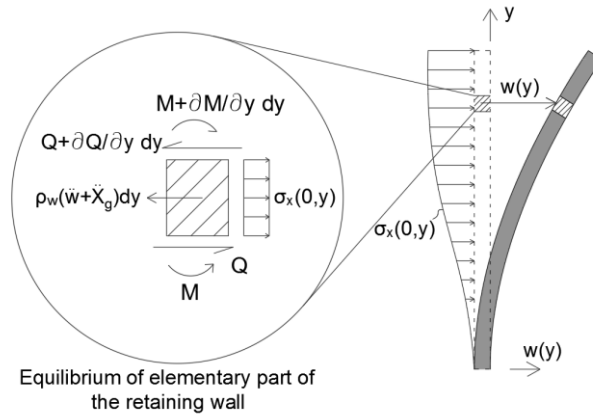


Figure 2. Equilibrium of forces for the retaining wall

## 3. PROPOSED SOLUTION

The pair of Equations 13 and 14 is uncoupled and, thus, can be solved in by substituting the former into the latter, to get

$$EI^* w^{(4)}(y) + w(y) \left( \frac{2}{1-\nu} G^* m - \omega^2 \rho_w \right) = \left( \omega^2 \rho_w - \left( \frac{2}{1-\nu} \right) \frac{\mathcal{L}}{a_{oc}^2 - a_0^2} H^2 \rho m \Phi(y) \right) \ddot{X}_g \quad (15)$$

Equation 15 is a non-homogeneous, fourth-order ordinary differential equation with constant coefficients, which can be solved in a straightforward manner once the function  $\Phi(y)$  has been established. Upon substitution of Equation (11) for  $\Phi$ , the general solution for the wall deflection is:

$$w(y) = C_1 \cos(\beta y) + C_2 \sin(\beta y) + C_3 \cosh(\beta y) + C_4 \sinh(\beta y) + \frac{\omega^2 \rho_w / EI^* - \Lambda_2 \Phi(y)}{(\pi/2)^4 + \Lambda_1} \quad (16)$$

where

$$\beta^4 = \frac{1}{4 EI^*} \left( \frac{2}{1-\nu} G^* m - \omega^2 \rho_w \right) \quad (17a)$$

$$\Lambda_1 = \frac{H^4}{EI^*} \left( \frac{2}{1-\nu} G^* m - \omega^2 \rho_w \right) \quad (17b)$$

$$\Lambda_2 = \left( \frac{2}{1-\nu} \right) \frac{\mathcal{L}}{a_{0c}^2 - a_0^2} \frac{H^6 \rho m \ddot{X}_g}{EI^*} \quad (17c)$$

are dimensionless solution parameters. The integration constants  $C_1, C_2, C_3, C_4$  are determined by enforcing the following boundary conditions at the top and bottom of the wall:

$$w(0) = 0 \quad (18a)$$

$$EI^* w^{(2)}(0) = K_R w^{(1)}(0) \quad (18b)$$

$$w^{(2)}(H) = 0 \quad (18c)$$

$$w^{(3)}(H) = 0 \quad (18d)$$

which correspond, respectively, to the zero wall-soil relative displacement at the base, the equilibrium of moment at the base, and the cantilever conditions at the top of the wall (zero shear force and moment). Note that the rotational spring  $K_R$  represents the rotational stiffness of the wall foundation (Mushkelishvili 1954, Dobry & Gazetas 1986). For a top-hinged wall, Equation 18d should be replaced by a condition of zero soil-wall relative displacement i.e.

$$w(H) = 0 \quad (18e)$$

Based on the above analysis, the total shear force  $Q_b$  and the base moment  $M_b$  on a cantilever wall can be obtained from Equation 13, by integrating the normal tractions along the wall height. This leads to the expressions

$$Q_b = -\frac{2}{1-\nu} G^* m \left( \frac{\mathcal{L}}{a_{0c}^2 - a_0^2} \left( \frac{H}{V_s^*} \right)^2 \ddot{X}_g \int_0^H \Phi(y) dy + \int_0^H w(y) dy \right) \quad (19a)$$

$$M_b = -\frac{2}{1-\nu} G^* m \left( \frac{\mathcal{L}}{a_{0c}^2 - a_0^2} \left( \frac{H}{V_s^*} \right)^2 \ddot{X}_g \int_0^H y \Phi(y) dy + \int_0^H y w(y) dy \right) \quad (19b)$$

Finally, the point of application of the total soil thrust,  $h$ , is equal to the ratio of the base moment  $M_b$  to the shear force  $Q_b$ .

For a top-hinged wall, the problem is statically indeterminate and equilibrium Equations (19) for the base shear and the overturning moment cease to be applicable. Instead,  $Q_b$  and  $M_b$  can be determined from beam theory as  $EI^* w^{(3)}(H)$  and  $EI^* w^{(2)}(H)$ , respectively, which account for equilibrium and compatibility of deformations. Following Younan & Veletsos (2000) for a better interpretation of the solution, the following parameters are defined:

The “impairment factor” of dynamic thrust as a result of wall flexibility and frequency:

$$I_f = \frac{Q_b^{flexible}}{Q_b^{rigid}} = \frac{Q_b(d_w)}{Q_b(d_w \rightarrow 0, d_\theta \rightarrow 0)} \quad (20a)$$

The dynamic amplification factor of soil thrust as a function of frequency:

$$AF = \frac{Q_b^{dynamic}}{Q_b^{static}} = \frac{Q_b(\omega)}{Q_b(\omega \rightarrow 0)} \quad (20b)$$

In the above equations,  $d_w$  and  $d_\theta$  denote the familiar dimensionless ratios

$$d_w = \frac{GH^3}{EI} \quad (21a)$$

$$d_\theta = \frac{GH^2}{K_R} \quad (21b)$$

#### 4. PSEUDOSTATIC RESPONSE

Younan and Veletsos (2000) derived a complex analytical solution to an analogous problem using Lagrange's equations in conjunction with pertinent eigenvalue expansions of the horizontal displacement over the vertical coordinate. A comparison of the proposed simplified solution against the more rigorous one by Younan and Veletsos is presented in Figure 3, in terms of normalized pseudo-static contact pressures (obtained at  $\omega = 0$ ) versus depth, for different values of normalized wall flexibility ( $d_w = 1, 5$  and  $40$ ) and a fixed-base cantilever retaining wall ( $d_\theta = 0$ ). Note that  $d_w$  usually varies between 10 and 20 for most walls. Also shown in the graph are results from a PLAXIS 2D Finite-Element (F.E.) analysis (Koutsantonakis 2017). Evidently, the discrepancy between the results of the two methods is quite small except near the wall base where the proposed solution underestimates the contact pressures. This is anticipated, as the proposed method converts the elasticity equations into an equivalent Winkler model in which the dynamic pressures are proportional to the difference between wall deflection and free-field soil displacement, which is exactly zero at the base of the wall. Also, the model predicts tensile dynamic pressures near the top of the wall with increasing wall flexibility - an anticipated interaction pattern between a flexural and a shear system. These tensile pressures are less pronounced in the proposed model than in the F.E. and Younan & Veletsos solutions, a trait which can be attributed to the number of modes involved in the analysis. These tensile stresses have the effect of lowering the height  $h$  of application of the total base shear force, as shown in Table 1 and Figure 4(a). It should be noticed that the overall seismic thrusts (area of the dynamic pressures in  $\sigma$ - $y$  space, as given in Table 2) is quite similar in the three solutions, especially for low  $d_w$ 's, as the proposed solution tends to overestimate the soil pressures near the top of the wall and underestimate them near the bottom.

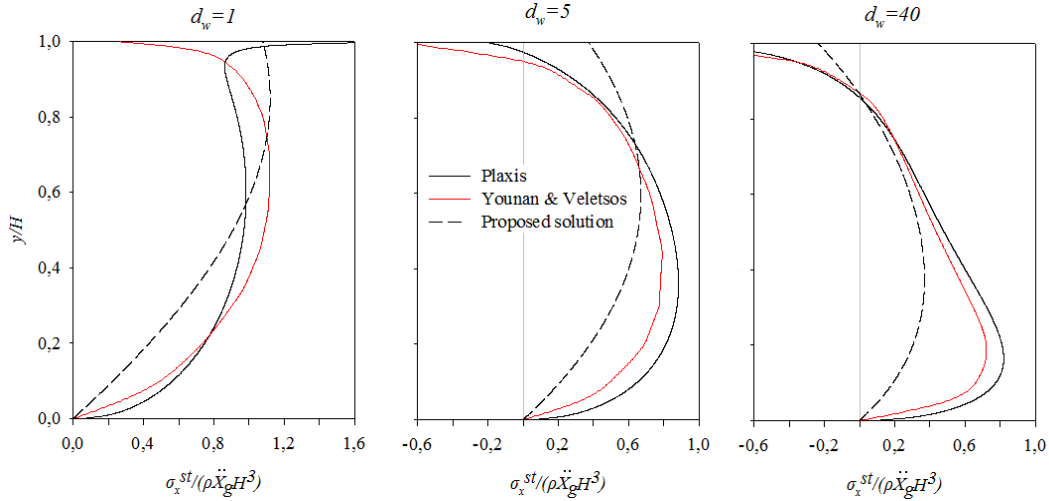


Figure 3. Normalized pseudo-static thrust against normalized depth for a fixed-base cantilever wall,  $\nu=1/3$ ,  $d_\theta=0$ .

The Younan & Veletsos solution considers - through pertinent eigen-expansions -  $J$  natural modes for the flexible wall and  $N$  modes for the retained soil. As illustrated in Figure 4(b), the proposed solution is in good agreement with the Younan & Veletsos solution for the normalized pseudo-static wall deflection, using the first mode for the wall and the soil ( $J = N = 1$ ), and is equally close to the exact response obtained for  $J = 2$  and  $N = 5$ . Overall, there is a fair agreement for the pseudo-static response between the proposed method, the Younan & Veletsos solution and the numerical F.E. results.

Table 1. Normalized pseudostatic height ( $h/H$ ) of application of base shear force for a fixed-base cantilever wall.



$d_w$	Younan and Veletsos (2000)	Proposed solution	F.E.M. (Plaxis 2D)
1	0.553	0.618	0.547
5	0.443	0.560	0.445
40	0.259	0.371	0.256

Table 2. Normalized pseudostatic base shear force  $Q_b/(\rho\ddot{X}_g H^2)$  for a fixed-base cantilever wall.

$d_w$	Younan and Veletsos (2000)	Proposed solution	F.E.M. (Plaxis 2D)
1	0.753	0.785	0.849
5	0.546	0.501	0.648
40	0.351	0.221	0.388

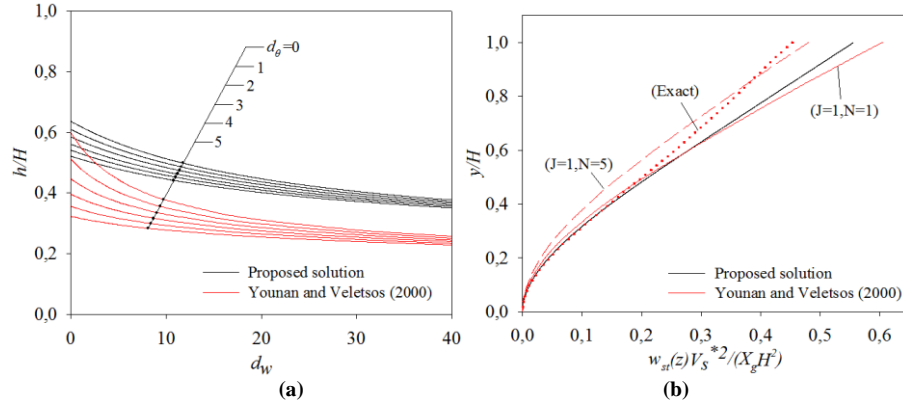


Figure 4. (a) Normalized height of application of pseudostatic soil thrust for a cantilever wall against normalized wall flexibility  $d_w$  and rotation compliance  $d_\theta$ , (b) normalized pseudostatic wall deflection for a flexible wall having  $d_w = 20$ .

## 5. DYNAMIC RESPONSE

The influence of frequency on the dynamic response of the system is shown in Figure 5 in terms of the normalized base shear  $Q_b$  and the amplification factor  $AF$ .  $f_1 (= V_s/4H)$  in the abscissa of the graphs is the fundamental natural frequency of the homogeneous soil stratum in the far field.

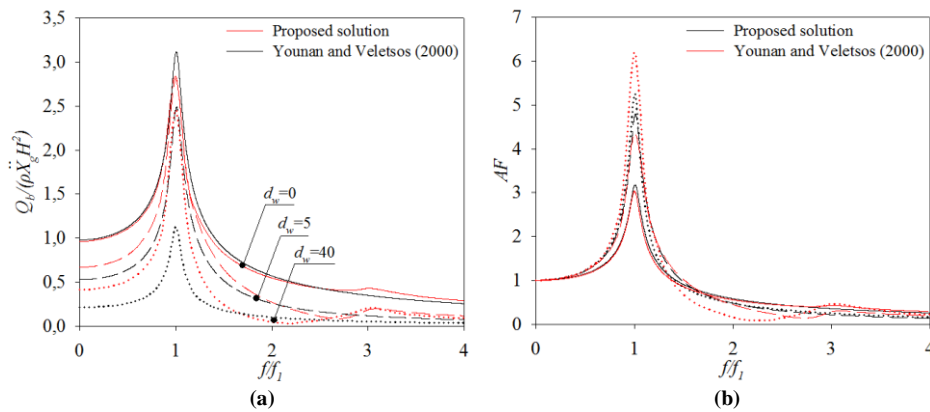


Figure 5. (a) Normalized dynamic soil thrust for a cantilever fixed-base wall, (b) Amplification factor against normalized frequency,  $v=1/3$ ,  $\delta = 10\%$ ,  $\delta_w = 4\%$

As observed in Figure 5a, the methods are in reasonably good agreement for  $d_w < 5$ , which covers a wide range of cases of practical importance. For  $d_w = 40$ , which lies outside the range of practical interest, the discrepancies are more pronounced, especially near resonance ( $f/f_i=1$ ). On the other hand, the comparison in terms of the amplification factor  $AF$  is more satisfactory, exhibiting deviations of less than 20% even for high wall flexibilities ( $d_w = 40$ ). The strong dependence of earth thrust on  $d_w$  (Figure 5a) elucidates the inability of classical limit-equilibrium methods to predict the soil thrust considering solely body forces acting on a soil prism behind the wall.

Figure 6a depicts the normalized base shear force for a cantilever wall against normalized excitation frequency  $f/f_i$  and wall flexibility  $d_w$ . Figure 6b depicts the impairment factor  $I_f$  (Equation 20a) referring to the reduction in total shear force  $Q_b$  due to wall flexibility.

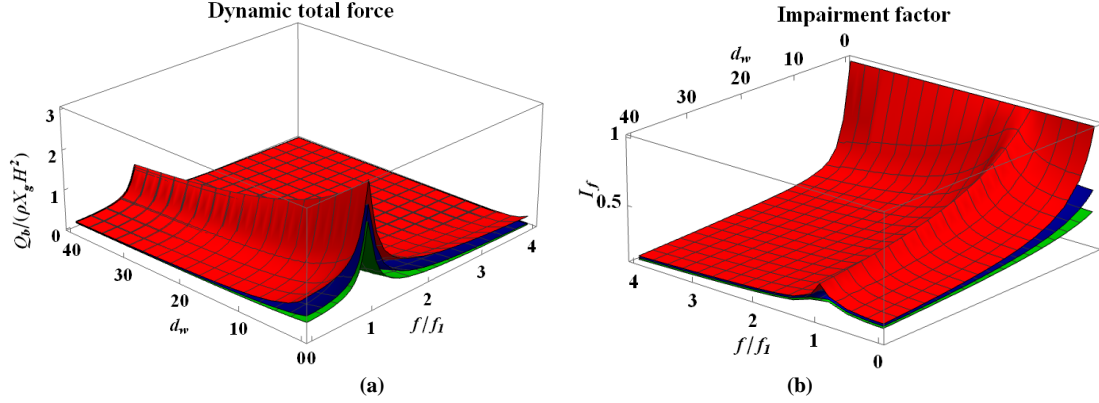


Figure 6. (a) Normalized dynamic soil thrust for a cantilever wall, (b) Impairment factor;  $d_\theta = 0$  (red plot),  $d_\theta = 1$  (blue plot),  $d_\theta = 2$  (green plot);  $\nu = 1/3$ ,  $\delta = 10\%$ ,  $\delta_w = 4\%$

As shown in Figure 6, wall flexibility  $d_w$  has a major influence on the magnitude of earth thrusts, while the rotational compliance  $d_\theta$  is of secondary importance. Figure 6b indicates that even a small value of wall flexibility ( $d_w < 5$ ) provides an enormous impairment (over 50%) at resonance. Higher wall flexibilities ( $d_w > 20$ ) lead to thrust reductions up to 80%. Note that for a rigid wall ( $d_w = 0$ ), the impairment factor reflects only the influence of foundation flexibility (rotational compliance  $K_R$ ), which leads to a decrease in thrust of about 20-40%.

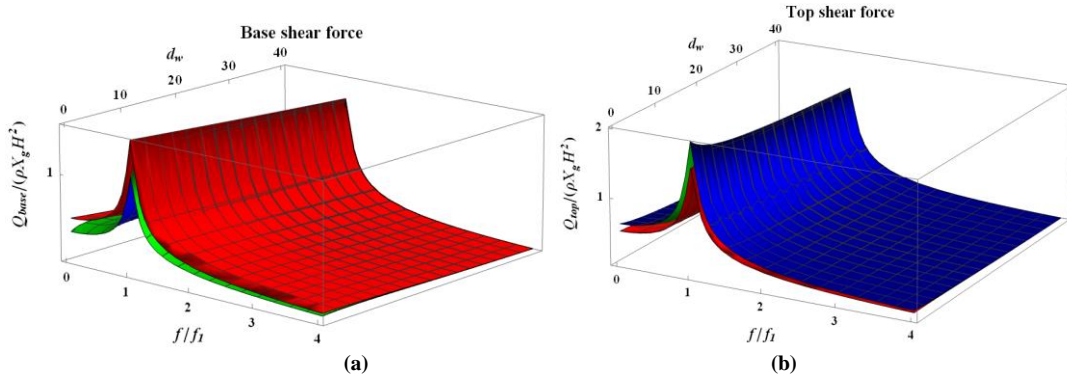


Figure 7. Normalized soil thrust for a top-hinged wall against normalized frequency and wall flexibility. (a) base shear and (b) hinge reaction at the top;  $d_\theta = 0$  (red plot),  $d_\theta = 1$  (blue plot),  $d_\theta = 2$  (green plot);  $\nu = 1/3$ ,  $\delta = 10\%$ ,  $\delta_w = 4\%$

Figure 7a illustrates the base shear  $Q_b$  force for a top-hinged (“propped”) cantilever wall and Fig 7b presents the reaction force at the top  $Q_{top}$  against normalized frequency and wall flexibility. Evidently, the prop may attract a higher seismic thrust than the base. For high wall flexibilities ( $d_w > 30$ ), the prop force  $Q_{top}$  is of the same magnitude as the base shear and insensitive to wall flexibility  $d_w$ . For high foundation compliances ( $d_\theta = 2$ ), the wall tends to respond as a hinged-hinged beam, and the rotational

spring  $K_R$  does not contribute considerably to the stiffness of the system. It is worth noting that the resonance frequency of the system is hardly affected by wall flexibility and takes place near  $f/f_1=1$  in the whole range of the parameters examined (Figures 5, 6, 7).

## 6. EQUIVALENT WINKLER SPRINGS

Rearranging terms in Equation 13, leads to a Winkler formulation that relates dynamic thrusts on the wall to the difference between wall deflection and free field displacement. This dependence can be cast in the following compact form:

$$\sigma_x(0, y) = k_y^i [u_{ff}(y) - w(y)] \quad (22)$$

where,

$$u_{ff}(y) = X(x \rightarrow \infty) \Phi(y) = -\frac{\mathcal{L} H^2}{(a_{oc}^2 - a_o^2) v_s^{*2}} \Phi(y) \ddot{X}_g \quad (23)$$

$$k_y^i = \frac{2}{\sqrt{(1-\nu)(2-\nu)}} (a_{oc}^2 - a_o^2)^{1/2} G^*/H \quad (24)$$

$u_{ff}$  is the free-field displacement developing at an infinite distance from the wall and  $k_y^i$  is a depth-independent Winkler spring coefficient intensity having dimensions of Force/Length<sup>3</sup>. The expressions in Equations (23) and (24) are the same as in Kloukinas et al. (2012) and are independent of wall flexibility. Corresponding solutions for rigid and flexible walls in inhomogeneous soil are available, respectively, in Brandenburg et al (2017) and in Durante et al (2018), which possess a number of advantages over the one presented in this work as they do not depend on the far-field displacement function  $\Phi(y)$ . It should be noted that using a shape function  $\Phi(y)$  that approximates better the wall deflection  $w(y)$  in Equation (16), is expected to provide superior values for  $k_y^i$ , like the ones reported in Durante et al (2018). However, that function will naturally deviate from the corresponding free-field solution and, thereby, will not reproduce the natural frequency of the soil layer. Evidently, a solution based on a single shape function,  $\Phi(y)$ , like the one at hand, cannot work in a satisfactory manner both near and away from the wall.

## 7. CONCLUSIONS

A simplified analytical solution was developed for the seismic response of flexible walls retaining a horizontal viscoelastic soil layer of constant thickness and material properties. The wall is modelled as an Euler-Bernoulli beam with either cantilever or top-hinged conditions pertaining to basement walls and bridge abutments. Under the assumption of zero vertical dynamic normal stresses, a modified Vlasov-Leontiev model was adopted which simplifies the analysis by suppressing the vertical spatial coordinate, thus reducing the independent variables from two to one (horizontal distance from the wall) leading to an elementary governing equation which is amenable to elementary treatment. The main conclusions of the study are:

- (1) Results from a pseudo-static analysis ( $\omega = 0$ ) compare favorably with corresponding results from the solution of Younan and Veletsos (2000) and a PLAXIS (2D) finite-element solution.
- (2) A parametric investigation of the dynamic response of the system shows that wall flexibility has a major influence in soil-structure interaction, by strongly reducing the mobilized soil thrust. On the other hand, foundation flexibility has a smaller effect in reducing the dynamic soil thrust.
- (3) For top-hinged (“propped”) walls, the support at the top greatly relieves the shear force developing at the base. Also, for  $d_0 > 2$  the rotational compliance of the wall foundation does

not influence the resistance of the support system which behaves as a simply supported beam. All the above are ignored by limit-equilibrium solutions recommended in seismic codes.

- (4) Following up on the Conclusions in 3 and 4 above, it seems that a base flexibility condition that renders a certain amount of displacement at the top of the wall may result in a pressure solution that is similar to a flexible wall solution with a similar top displacement. It might be possible that the parameter space covered for base flexibility is somehow incompatible with the parameter space covered by wall flexibility, thereby resulting in the appearance that wall flexibility is more important. This possibility will be investigated in a follow-up paper.
- (5) An equivalent Winkler model was developed for the conditions at hand resulting in the same expression as that derived by Kloukinas et al. (2012) for the case of a rigid wall. Using a function  $\Phi(y)$  that approximates better the wall deflection  $w(z)$ , would have provided superior values for  $k_y$  like the ones reported in Durante et al (2018). However, that function won't reproduce the natural frequency of the soil layer as it will naturally deviate from the free-field solution. Evidently, a solution based on a single shape function,  $\Phi(y)$ , like the one at hand, cannot work in a satisfactory manner both near and away from the wall.

As a final remark, it is worth noting that the above effects are inherently of kinematic nature, as they stem from displacement mismatch between wall and soil, which is ignored in classical limit-equilibrium solutions such as the M-O formulae. In light of the advantages of the elastodynamic solutions, the M-O type methods should gradually reach retirement.

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